MULTIMEDIA UNIVERSITY

STUDENT ID NO

MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 2, 2016/2017

ETN 3096 - DIGITAL SIGNAL PROCESSING

(All sections / Groups)

1 MARCH 2017
2.30 pm – 4.30 pm
(2 Hours)

INSTRUCTION TO STUDENTS

1. This question paper consists of 4 pages (including cover page) with 4 questions and 6-page list of formulas given in the appendix.

2. Attempt all **FOUR** questions. The distribution of the marks for each question is given. The overall total marks is 60.

3. Please write all your answers in the answer Booklet provided.

QUESTION 1

a) Briefly discuss the role of an anti-aliasing filter used in a digital signal processing system.

[2 marks]

b) A linear time invariant system (LTI) is defined by the impulse response $h(n)$. Compute the output of the LTI system given the input $x(n)$.

$$x(n) = [\underline{2},\ 1,\ 1], \qquad h(n) = [\underline{1},\ 2,\ 3,\ 4]$$

Note that the underlined numbers represent samples value at $n = 0$.

[4 marks]

c) The averaging filter is commonly used to smooth a noisy signal. The output $y(n)$ of an averaging filter is given as follow

$$y(n) = 1/3\ (\ x(n) + x(n\ \text{-}1) + x(n\text{-}2)\ )$$

i) Determine if the averaging filter is time invariant and causal. Give proof or justification to support your answer.

[3 marks]

ii) What is the impulse response of the system? Plot the impulse response sequence $h(n)$.

[2 marks]

iii) Determine the Discrete Time Fourier Transform of the impulse response of system in c(ii).

[4 marks]

Continued..

QUESTION 2

a) A system is described by the following difference equation. The expression $u(n)$ is the unit step function.

$$y(n)-0.5y(n-1)=2(0.8)^n u(n)$$

i) Given that $Y(z)$ is the z-transform of $y(n)$, Determine $Y(z)$ and it's causal solution when the initial condition is $y(-1) = 2$.

[5 marks]

ii) Is the solution $y(n)$ obtained in (a)(i) stable? Discuss and gives justification.

[2 marks]

b) Given a sequence $x(n) = [\ \underline{1}\ 2\ 2\ 5\ ]$, evaluate the 4 point Discrete Fourier Transform (DFT) of the sequence $x(n)$. Compute all four DFT samples.

[4 Marks]

c) Assume $X(k)$ are the DFT samples of the sequence $x(n)$. The sampling frequency used is 1000 Hz. A total of 200 samples $\{x(0), x(1), \ldots x(199)\}$ has been gathered.

i) Find the sampling period, time index and actual time for the sample $x(8)$ (signal in time).
[1 mark]

ii) Determine the frequency resolution and the mapped frequency (in Hz) for the DFT sample $X(5)$.

[1 mark]]

iii) Propose a spectrum estimation method using the DFT that provides a frequency resolution of 1 Hz. The sampling frequency remains the same. What is the benefit of using higher frequency resolution?

[2 marks]

Continued..

QUESTION 3

a) You are required to design a 3-tap (order=2) FIR lowpass filter with a cut-off frequency of 1,500 Hz and a sampling rate of 8,000 samples/sec. Determine the filter coefficients, transfer function, and difference equation of the filter using a Hamming window function.

[16 marks]

b) Explain the design procedure of bilinear z-transformation for designing an IIR filter.

[4 marks]

QUESTION 4

a) The transfer function of a digital filter is given as $H(z)$. Implement the realization of the system based on the following structure.

$$H(z) = \frac{3(2z^2 + 5z + 4)}{(2z^2 + 5z + 2)}$$

i) Direct form 1

[2 marks]

ii) Direct form 2

[2 marks]

iii) Parallel structure with first order section

[4 marks]

iv) Comment on the implementation complexity of the three structures proposed in (a) (i), (ii) and (iii). Which structure give the most complex implementation and which one gives the least complex implementation? The complexity is measured based on number of additions, multiplications and memory cells used.

[2 marks]

End of paper

APPENDIX

Table of z-Transform Pairs

$x(n), n \geq 0$	z-Transform, $X(z)$
$\delta(n)$	1
$au(n)$	$\dfrac{az}{z-1}$
$nu(n)$	$\dfrac{z}{(z-1)^2}$
$n^2u(n)$	$\dfrac{z(z+1)}{(z-1)^3}$
$a^n u(n)$	$\dfrac{z}{z-a}$
$e^{-an}u(n)$	$\dfrac{z}{z-e^{-a}}$
$na^n u(n)$	$\dfrac{az}{(z-a)^2}$

Properties of the z-Transform

Suppose $X(z) = Z\left(x(n)\right), X_1(z) = Z\left(x_1(n)\right)$, and $X_2(z) = Z\left(x_2(n)\right)$

Linearity: If $x(n) = ax_1(n) + bx_2(n)$, then $X(z) = aX_1(z) + bX_2(z)$

Linear Convolution: If $x(n) = x_1(n) * x_2(n)$, then $X(z) = X_1(z) \cdot X_2(z)$

$Z\left(x(n-m)\right) = x(-m) + x(-m+1)z^{-1} + x(-m+2)z^{-2} + \ldots + x(-1)z^{-(m-1)} + z^{-m}X(z)$

If all initial conditions are zero, then $Z\left(x(n-m)\right) = z^{-m}X(z)$ (Shift theorem)

Discrete Time Fourier Transform and its Inverse

$$X(\Omega) = \sum_{n=-\infty}^{+\infty} x[n]e^{-j\Omega n}$$

$$x(n) = \frac{1}{2\pi}\int_0^{2\pi} X(\Omega)e^{j\Omega n}d\Omega$$

Summary of Ideal Impulse Responses for Standard FIR Filters

Filter Type	Ideal Impulse Response $h(n)$
Lowpass:	$h(n) = \begin{cases} \dfrac{\Omega_c}{\pi}, & n = 0 \\ \dfrac{\sin(\Omega_c n)}{n\pi}, & n \neq 0 \end{cases}$
Highpass:	$h(n) = \begin{cases} \dfrac{\pi - \Omega_c}{\pi}, & n = 0 \\ \dfrac{-\sin(\Omega_c n)}{n\pi}, & n \neq 0 \end{cases}$
Bandpass:	$h(n) = \begin{cases} \dfrac{\Omega_H - \Omega_L}{\pi}, & n = 0 \\ \dfrac{\sin(\Omega_H n) - \sin(\Omega_L n)}{n\pi}, & n \neq 0 \end{cases}$
Bandstop:	$h(n) = \begin{cases} \dfrac{\pi - (\Omega_H - \Omega_L)}{\pi}, & n = 0 \\ \dfrac{-\sin(\Omega_H n) + \sin(\Omega_L n)}{n\pi}, & n \neq 0 \end{cases}$

FIR Filter Length Estimation Using Window Functions

Window Type	**Window Function** $w(n), -M \leq n \leq M$
Rectangular	1
Hanning	$0.5 + 0.5\cos\left(\dfrac{n\pi}{M}\right)$
Hamming	$0.54 + 0.46\cos\left(\dfrac{n\pi}{M}\right)$
Blackman	$0.42 + 0.5\cos\left(\dfrac{n\pi}{M}\right) + 0.08\cos\left(\dfrac{2n\pi}{M}\right)$

FIR Filter Length Estimation Using Window Functions

Normalized Transition Width $\Delta f = \dfrac{\lvert f_{stop} - f_{pass} \rvert}{f_{sampling}}$			
Type of Window	**Window Length N**	**Stopband Attenuation, $20\log_{10}\delta_s$ (dB)**	**Passband Ripple**
Rectangular	$N = 0.9/\Delta f$	21	0.7416
Hanning	$N = 3.1/\Delta f$	44	0.0546
Hamming	$N = 3.3/\Delta f$	53	0.0194
Blackman	$N = 5.5/\Delta f$	74	0.0017

Bilinear Transformation (BLT)

1. Frequency prewarping

Let ω_a denote the analog frequency marked on the $j\omega$-axis on the s-plane, and ω_d denote the digital frequency marked on the unit circle in the z-plane.

For the lowpass filter and highpass filter:

$$\omega_a = \frac{2}{T}\tan\left(\frac{\omega_d T}{2}\right)$$

For the bandpass filter and bandstop filter:

$$\omega_{al} = \frac{2}{T}\tan\left(\frac{\omega_l T}{2}\right), \quad \omega_{ah} = \frac{2}{T}\tan\left(\frac{\omega_h T}{2}\right)$$

and $\omega_0 = \sqrt{\omega_{al}\omega_{ah}}, W = \omega_{ah} - \omega_{al}$.

2. Prototype transformation using the lowpass prototype $H_P(s)$

From lowpass to lowpass: $H(s) = H_P(s)\big|_{s=\frac{s}{\omega_a}}$

From lowpass to highpass: $H(s) = H_P(s)\big|_{s=\frac{\omega_a}{s}}$

From lowpass to bandpass: $H(s) = H_P(s)|_{s=\frac{s^2+\omega_0^2}{sW}}$

From lowpass to bandstop: $H(s) = H_P(s)|_{s=\frac{sW}{s^2+\omega_0^2}}$

where ω_a denotes the analog frequency, $\omega_0 = \sqrt{\omega_{al}\omega_{ah}}, W = \omega_{ah} - \omega_{al}$.

3. Substitute the BLT to obtain the digital filter

$$H(z) = H(s)|_{s=\frac{2}{T}\cdot\frac{z-1}{z+1}}$$

Conversion from Analog Filter Specifications to Lowpass Prototype Specifications

Analog Filter Specifications	**Lowpass Prototype Specifications**
Lowpass: ω_{ap}, ω_{as}	$v_s = \frac{\omega_{as}}{\omega_{ap}}$
Highpass: ω_{ap}, ω_{as}	$v_s = \frac{\omega_{ap}}{\omega_{as}}$
Bandpass: $\omega_{apl}, \omega_{aph}, \omega_{asl}, \omega_{ash}$ $\omega_0 = \sqrt{\omega_{apl}\omega_{aph}}, \omega_0 = \sqrt{\omega_{asl}\omega_{ash}}$	$v_s = \frac{\omega_{ash} - \omega_{asl}}{\omega_{aph} - \omega_{apl}}$
Bandstop: $\omega_{apl}, \omega_{aph}, \omega_{asl}, \omega_{ash}$ $\omega_0 = \sqrt{\omega_{apl}\omega_{aph}}, \omega_0 = \sqrt{\omega_{asl}\omega_{ash}}$	$v_s = \frac{\omega_{aph} - \omega_{apl}}{\omega_{ash} - \omega_{asl}}$

ω_{ap}, passband frequency edge; ω_{as}, stopband frequency edge

ω_{apl}, lower cutoff frequency in passband; ω_{aph}, upper cutoff frequency in passband

ω_{asl}, lower cutoff frequency in stopband; ω_{ash}, upper cutoff frequency in stopband

ω_0, geometric center frequency

Digital Butterworth and Chebyshev Filter Designs

With the given passband ripple A_p dB at the normalized passband frequency edge $v_p = 1$, and the stopband attenuation A_s dB at the normalized stopband frequency edge v_s, we determine the lowpass prototype order as

$$\varepsilon^2 = 10^{0.1A_p} - 1$$

The **Butterworth** lowpass prototype with order n is given as

$$n \geq \frac{\log_{10}\left(\frac{10^{0.1A_s} - 1}{\varepsilon^2}\right)}{|2\log_{10}(v_s)|}$$

where ε is the absolute ripple specification.

The **Chebyshev** lowpass prototype with order n is given as

$$n \geq \frac{\cosh^{-1}\left(\sqrt{\dfrac{10^{0.1A_s}-1}{\varepsilon^2}}\right)}{\cosh^{-1}(v_s)}$$

where $\cosh^{-1}(x) = \ln\left(x + \sqrt{x^2 - 1}\right)$.

3-dB Butterworth Lowpass Prototype Transfer Functions ($\varepsilon = 1$)

n	$H_P(s)$
1	$\dfrac{1}{s+1}$
2	$\dfrac{1}{s^2+1.4142s+1}$
3	$\dfrac{1}{s^3+2s^2+2s+1}$

Chebyshev Lowpass Prototype Transfer Functions with 1 dB Ripple ($\varepsilon = 0.5088$)

n	$H_P(s)$
1	$\dfrac{1.9652}{s+1.9652}$
2	$\dfrac{0.9826}{s^2+1.0977s+1.1025}$
3	$\dfrac{0.4913}{s^3+0.9883s^2+1.2384s+0.4913}$

CLOSED-FORM EXPRESSION FOR SOME USEFUL SERIES

$\sum_{n=0}^{N-1} a^n = \dfrac{1-a^N}{1-a}$	$\sum_{n=0}^{\infty} na^n = \dfrac{a}{(1-a)^2}$
$\sum_{n=0}^{N-1} na^n = \dfrac{(N-1)a^{N+1} - Na^N + a}{(1-a)^2}$	$\sum_{n=0}^{N-1} n^2 = \dfrac{1}{6}N(N-1)(2N-1)$
$\sum_{n=0}^{N-1} n = \dfrac{1}{2}N(N-1)$	$\sum_{n=N_1}^{N_2} a^n = \dfrac{a^{N_1-1} - a^{N_2}}{1-a}$
$\sum_{n=0}^{\infty} a^n = \dfrac{1}{1-a}$ $\quad |a| < 1$	

Discrete-time Fourier Analysis

The discrete Fourier transform (DFT)

$$X[k] = \sum_{n=0}^{N-1} x[n]e^{-j2\pi kn/N}, \quad k = 0, 1, \ldots, N-1$$

Properties of the DFT

Property	$x[n]$	$X[k]$
Linearity	$A_1x_1[n] + A_2x_2[n]$	$A_1X_1[k] + A_2X_2[k]$
Time shifting	$x[\langle n - n_0\rangle_N]$	$X[k]W_N^{kn_0}$
Frequency shifting	$x[n]W_N^{-k_0n}$	$X[\langle k - k_0\rangle_N]$
Time reversal	$x[\langle -n\rangle_N]$	$X[\langle -k\rangle_N]$
Conjugation	$x^*[n]$	$X^*[\langle -k\rangle_N]$
Convolution	$x[n] \circledast y[n]$	$X[k]Y[k]$
Modulation	$Nx[n]y[n]$	$X[k] \circledast Y[k]$

The decimation-in-time FFT